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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- LM Landing Point Flexibility Provided
by the Lunar Flying Unit on Single
Launch Lunar Missions

TM- 68-2015-3**DATE-** April 16, 1968**FILING CASE NO(S)-** 340**AUTHOR(S)-** D. R. Valley

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ABSTRACT

Post Apollo mission planning has generally implied a greater LM landing accuracy for missions to specific sites on the lunar surface. This memorandum investigates the application of Lunar Flying Unit (LFU) "multi-hop" sorties to this type of mission so as to create a better understanding of their influence on the landing precision required.

The main body of the memorandum describes and illustrates a somewhat rigorous analysis technique for an example mission using an LFU for surface mobility. The end product of the analysis is a description of the LM landing point dispersion that can be tolerated without degradation of the LFU mission objectives. Appendix B develops a simpler, approximate method for obtaining the same type results.

Finally, Appendix C applies the analysis techniques to an actual mission proposed for the Hadley Rille region of the moon. The results clearly illustrate the concept of an allowable LM landing area that is adjustable, and lead to some logical planning changes to the LFU mission. Incorporation of these changes is then shown to increase the landable terrain available for the LM touchdown without sacrifice to the LFU mission objectives.

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FLEXIBILITY PROVIDED BY THE LUNAR FLYING
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(Bellcomm, Inc.) 37 p

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1100 Seventeenth Street, N.W. Washington, D. C. 20036

SUBJECT: LM Landing Point Flexibility Provided by the Lunar Flying Unit on Single Launch Lunar Missions - Case 340

DATE: April 16, 1968

FROM: D. R. Valley

TM-68-2015-3

TECHNICAL MEMORANDUM

INTRODUCTION

Post Apollo lunar mission planning has generally implied a greater LM landing accuracy for missions to specific sites on the lunar surface. This memorandum investigates application of the Lunar Flying Unit (LFU) to this type of mission so as to create a better understanding of its influence on the necessary LM landing precision. A method for determining the tolerable LM landing point dispersion is described in detail and applied to a hypothetical mission to illustrate the analytical procedures. To get a little closer to reality, a recently proposed mission⁽¹⁾ to the Hadley Rille region of the moon has been included in Appendix C. This mission involves three "multi-hop" LFU sorties to visit seven (7) specific locations (sites*) selected for scientific reasons. An analysis of this mission by the methods described in this memorandum is included to demonstrate the type of results obtainable. In addition, the same procedures have been applied to some alternate schemes for conducting the LFU sorties to these same 7 locations. The results clearly indicate the concept of adjusting the allowable LM landing area through careful planning of the LFU sorties.

GENERAL BACKGROUND

Landing errors by the LM generally infer increased distances and traverse times to reach specific locations on the lunar surface. The extent of the tolerable landing error, therefore, is primarily a function of the surface traversing capability with respect to both time and distance. Perhaps the most significant single advantage of the flying unit concept is the relatively short travel time required. By virtue of this advantage, the LFU effectively eliminates travel time as a serious consideration, and the allowable LM landing error becomes

⁽¹⁾ Hadley Rille Mission presented to the Group for Lunar Exploration (GLEP) at their February 26, 1968 meeting.

* The term "site" used throughout this memorandum refers to specific locations on the lunar surface to be visited with an LFU rather than its normal connotation as the LM landing location.

a function of the LFU's capability to negotiate the additional distances involved. The main purpose of the analysis is to determine how much landing point dispersion can be tolerated without degradation of the LFU mission objectives.

RESULTS

The analysis of the LFU sorties in the example mission show that a fairly wide LM landing point dispersion can be tolerated without compromising mission plans for a given sortie. While the actual size of the allowable landing footprint is highly dependent on specific sortie details (payloads and distances between selected sites), there are some general observations that can be made:

1. The allowable landing footprint will be approximately elliptical in shape.
2. The major axis of the ellipse will lie along a line connecting the first and the last site to be visited during the LFU sortie.

These observations, although not astounding, can prove very useful for planning the LFU sorties of a lunar mission designed to explore several pre-selected sites. The analytical procedures outlined later provide a means of generating an allowable LM landing area (footprint) from within which a planned "multi-hop" LFU sortie can be flown without exceeding prescribed propellant limitations. Separate landing footprints are generated for each planned LFU sortie and superimposed on a geographical layout for the total lunar mission. The overlap area between the individual footprints then represents the allowable landing point dispersion for the entire mission. From any point within this area, all the LFU sorties can be flown as planned without exceeding the flying unit's propellant constraints.

MISSION PLANNING ASPECTS

As mentioned previously, simply knowing the landing footprint's elliptical shape and the orientation of its major axis is of value for the first cut planning of LFU sorties. For example, the sequence of visiting the individual sites and the combinations of sites to be covered by each of the LFU sorties can be tentatively selected with this knowledge. The orientation of the allowable landing ellipses can thus be modified to find the combination of sortie plans that provide the greatest overlap area within the landable terrain surrounding the mission site. Payload weights to be carried to each site can then be incorporated into the analysis methods to determine the size of

the overlap area. The important aspect here is to realize that the positioning and size of the allowable LM landing area are adjustable. The sequence in which the selected sites are visited can be altered to change the footprint's position, and the payload weights can change its size. The effect of increasing LFU payloads is indicated on Figure 5. As more details for a given mission become known, the procedures described here can be used to plan a surface mission that offers the highest success probability. Appendix C includes an analysis of an actual mission proposal to demonstrate these techniques.

LIMITATIONS OF ANALYSIS

The example mission is oriented exclusively toward flying unit application from a site accessibility standpoint. It should be realized that considerations other than sheer accessibility could influence the selection of a particular LM landing point. For example, if payloads beyond the LFU's capability are desired at a given location, or if it is necessary that the LM be nearby for operational reasons, a greater landing accuracy would still be required.

The detailed analytical procedures described in the following sections should not infer untold accuracy of results because of their somewhat rigorous nature. The results are only as good as the basic LFU weights and performance assumed. The vehicle weights and payloads incorporated are generally accepted planning values being used today. The performance data (ΔV vs Range and I_{sp}) have been taken from information published by Bell Aerosystems⁽²⁾. On the strength of Bell Aerosystems' flight experience with a one-man flying vehicle, this data was selected over a variety of others available from theoretical flight trajectory calculation work.

MISSION ANALYSIS - GENERAL

A mission including two "multi-hop" sorties has been fabricated for analysis. Each of these LFU sorties will be analyzed and discussed individually and then the results combined to demonstrate a means of assessing the implications that the LFU's capabilities could have on mission planning.

BASIC ASSUMPTIONS

Since this memorandum deals with an evaluation of the LFU capability, the mission analysis will be confined to the details involving the flying unit sorties. The following basic assumptions have been incorporated:

(2) Flight Test of A One-Man Flying Vehicle Vol. II - Final Report - Mission Applications Studies - Bell Aerosystems Report No. 2330-950002, July 1967.

1. All lunar sites to be visited via the LFU are pre-planned locations to be visited in a fixed sequence with a specified payload weight carried to each site.
2. The LFU starts each sortie with full propellant tanks and is allowed to burn only 90% of the propellant load.
3. Flat-top type trajectories with a 150 ft/sec horizontal velocity were assumed for all LFU flights. The range vs ΔV data was taken from Reference 3 and is shown on Figure 6.
4. The following LFU weight and performance data were used:

Vehicle dry weight	180 lbs
Propellant loaded	300 lbs
Crew (suited astronaut)	370 lbs
Total Vehicle Weight (less payload)	850 lbs
Specific Impulse	285 sec

DETAILS OF EXAMPLE MISSION

Figure 1 illustrates the geographical layout of five specific lunar sites to be visited with the LFU during the mission. Two LFU sorties have been planned to cover these five locations. The first sortie (3-hops) visiting sites #1 and #2; and the second sortie (4-hops) going to the remaining three sites. The details of the two sorties are shown below:

SORTIE NO. 1 (3-hops)

<u>Hop</u>	<u>Traverse</u>	<u>Range</u>	<u>Payload</u>
1	*LM to Site 1	?	200 lbs
2	Site 1 to Site 2	2 n mi	100 lbs
3	Site 2 to *LM	?	50 lbs

*LM - indicates the LM landing point.

(3) Determination of Lunar Flying Vehicle Range and Payload Capabilities for Mission Planning Purposes, Bellcomm Memorandum for File, D. R. Valley, December 26, 1967.

SORTIE NO. 2 (4-hops)

<u>Hop</u>	<u>Traverse</u>	<u>Range</u>	<u>Payload</u>
1	*LM to Site 3	?	100 lbs
2	Site 3 to Site 4	2 n mi	75 lbs
3	Site 4 to Site 5	2 n mi	25 lbs
4	Site 5 to *LM	?	25 lbs

It should be noted that the flight range of the first and last hop of each sortie are not known since these distances depend on the location of the LM landing point. The main purpose of this analysis is to determine the possible landing point locations from which the planned sorties can be flown without burning more than 90% of the LFU propellant load.

ANALYSIS PROCEDURE

Since the LFU propellant is the limiting factor in determining the allowable landing point dispersion, it will first be necessary to relate the LFU propellant requirements to the flight range of each hop and the payload weights carried. The following expression developed in Appendix A represents this relationship:

$$\begin{aligned}
 wp_t = W \left(R_1 R_2 R_3 \dots R_n - 1 \right) + PL_1 \left(R_1 - 1 \right) + PL_2 \left(R_1 \right) \left(R_2 - 1 \right) \\
 + PL_3 \left(R_1 R_2 \right) \left(R_3 - 1 \right) + \dots + PL_n \left(R_1 R_2 R_3 \dots R_{n-1} \right) \left(R_n - 1 \right)
 \end{aligned}
 \tag{1}$$

where:

n = number of flying unit hops

wp_t = total weight of propellant burned

W = vehicle empty weight (including crew and propellant reserve)

$PL_1 - PL_n$ = payload weights carried on respective hops

$R_1 - R_n$ = vehicle mass ratios associated with the ΔV^* requirements of each hop $\left(R = \exp \frac{\Delta V}{g_o I_{sp}} \right)$

Although Equation (1) looks cumbersome, one should remember that for a planned "multi-hop" sortie, the flight ranges for first and last hops are the only unknown quantities. In Equation (1) for example, the only unknown quantities are R_1 and R_n . With this in mind, Equation (1) can be rearranged to show the relationship between R_1 and R_n ; or between the flight ranges of the first and last hop:

$$R_1 = \frac{W + wp_t + PL_1}{X} \quad (2)$$

where:

$$X = (W + PL_n)(R_2 R_3 \cdots R_n) + (PL_1 - PL_2) + R_2(PL_2 - PL_3) \\ + R_2 R_3(PL_3 - PL_4) + \cdots + (R_2 R_3 \cdots R_{n-1})(PL_{n-1} - PL_n)$$

With R_1 and R_n the only unknowns, Equation (2) can be reduced to:

$$R_1 = \frac{K_1}{K_2 R_n + K_3} \quad (3)$$

where:

$$K_1 = W + wp_t + PL_1$$

$$K_2 = (W + PL_n)(R_2 R_3 \cdots R_{n-1})$$

$$K_3 = (PL_1 - PL_2) + R_2(PL_2 - PL_3) + R_2 R_3 (PL_3 - PL_4) \\ + \cdots + (R_2 R_3 \cdots R_{n-1})(PL_{n-1} - PL_n)$$

* ΔV requirements are synonymous with flight range and can be obtained from the ΔV vs. Range data shown on Figure 6.

The values of K_1 , K_2 and K_3 are constant for a given sortie plan. Equation (3) relates the flight ranges of the first and last LFU hops to each other in terms of all the known sortie and flying vehicle parameters. The use of Equation (3) can best be illustrated by proceeding with the analysis of the first sortie of the example mission.

ANALYSIS OF SORTIE NO. 1 (3-hops)

From the planned sortie and flying unit parameters, Equation (2) with $n = 3$ becomes

$$R_1 = \frac{W + wp_t + PL_1}{\left\{ W_1 + PL_3 \right\} \left\{ R_2 R_3 \right\} + \left\{ PL_1 - PL_2 \right\} + R_2 \left\{ PL_2 - PL_3 \right\}}$$

(obtained by substituting $n = 3$ into Equation (2) and eliminating all terms containing a subscript higher than 3).

The known parameters are as follows:

$$W = 580 \text{ lbs (180 lb dry wt + 370 lb crew + 30 lb propellant reserve)}$$

$$wp_t = 270 \text{ lbs (90\% of propellant load)}$$

$$PL_1 = 200 \text{ lbs}$$

$$PL_2 = 100 \text{ lbs}$$

$$PL_3 = 50 \text{ lbs}$$

$$R_2 = \exp \frac{\Delta V_2}{g_0 I_{sp}} \text{ where } \Delta V_2 = \Delta V \text{ required for 2nd LFU hop}$$

(Site #1 to Site #2 = 2 n mi range) from Figure 6,
the ΔV for a 2 n mi range is 820 ft/sec.

$$\therefore R_2 = \exp \frac{820}{g_0 (285)}$$

Substituting these known quantities into the expressions for K_1 , K_2 and K_3 gives:

$$K_1 = W + w p_t + PL_1 = 580 + 270 + 200 = 1050$$

$$K_2 = (W + PL_3) R_2 = [580 + 50] \cdot \exp \frac{820}{g_o(285)} = 688.93$$

$$K_3 = (PL_1 - PL_2) + R_2 (PL_2 - PL_3)$$

$$= (200 - 100) + \left(\exp \frac{820}{g_o(285)} \right) (100 - 50) = 154.68$$

and Equation (3) becomes:

$$R_1 = \frac{K_1}{K_2 R_3 + K_3} = \frac{1050}{688.93 R_3 + 154.68} \quad (4)$$

The above equation is sufficient to describe the allowable LM landing area from which sortie No. 1 could be flown as planned without exceeding the 270 lb propellant limitation. The process for accurately determining the allowable LM landing area involves a geometric construction procedure illustrated on Figure 2. Assuming a distance between the last LFU stop (Site #2) and the LM landing point provides a value for R_3 by converting the assumed range to a ΔV requirement (ΔV_3) with Figure 6. Using this value of $R_3 \left(\exp \frac{\Delta V_3}{g_o I_{sp}} \right)$, the corresponding value of R_1 can be obtained from Equation (4). The value of R_1 , in turn, can be expressed as a ΔV ($\Delta V_1 = g_o I_{sp} \ln R_1$) and converted to a flight range with Figure 6. Thus for the assumed distance between the last LFU stop and the LM landing point, the computed value of R_1 provides the maximum allowable distance between the LM landing point and the first LFU stop (Site #1).

Figure 2 outlines and illustrates the geometric construction procedures for determining the allowable LM landing area. Two circles are drawn; one about the last LFU stop (Site #2) with a radius equal to the assumed distance and the second around the first LFU stop (Site #1) with a radius equal to the distance corresponding to the calculated value of R_1 . The intersections of the two circles represent landing point locations

from which the LFU will burn 270 lbs of propellant in completing the planned sortie. The process can be repeated by varying the assumed distance. The intersections of the circles will create a series of points which will be the locus of the allowable LM landing area from within which the LFU sortie can be flown without exceeding the propellant constraint.

If, in applying the above procedures, the two circles do not intersect, the flight range assumed is such that either:

- 1) The sortie cannot be completed within the allowable propellant limit because the assumed distance was too great; or
- 2) The sortie can be completed without burning all of the allowable propellant because the distance was too small.

The more points generated in the above fashion, the more accurately the allowable landing area can be described. Figure 3 indicates a complete trace of the allowable LM landing area for sortie No. 1.

RESULTS FOR SORTIE NO. 1

It can be seen from Figure 3 that a relatively wide LM landing point dispersion is possible. As long as the LM lands within the 5.5 x 6 n mi elliptically shaped area shown, the planned sortie can be flown. It should also be noted that the elliptically shaped area has its major axis on a line drawn through the first and last of the lunar sites visited by the LFU.

ANALYSIS OF SORTIE NO. 2 (4-hops)

The procedure followed for the second LFU sortie is exactly the same and will be only partially repeated to indicate the differences that occur in the application of Equation (2) when a 4-hop ($n=4$) sortie is involved. The relationship between the first and last hop ΔV 's (ranges) will be as follows:

$$R_1 = \frac{W + wp_t + PL_1}{(W + PL_4)(R_2 R_3 R_4) + (PL_1 - PL_2) + R_2(PL_2 - PL_3) + R_2 R_3(PL_3 - PL_4)}$$

The known parameters from the plan for sortie No. 2 are as follows:

$$W = 580 \text{ lbs (same as first sortie)}$$

$$wp_t = 270 \text{ lbs (same as first sortie)}$$

$$PL_1 = 100 \text{ lbs}$$

$$PL_2 = 75 \text{ lbs}$$

$$PL_3 = 25 \text{ lbs}$$

$$PL_4 = 25 \text{ lbs}$$

$$R_2 = \exp \frac{\Delta V_2}{g_o I_{sp}} \quad (\Delta V_2 \text{ corresponds to a flight range of } 2 \text{ n mi} - \text{distance between points 3 and 4})$$

$$R_3 = \exp \frac{\Delta V_3}{g_o I_{sp}} \quad (\Delta V_3 \text{ corresponds to a flight range of } 2 \text{ n mi} - \text{distance between points 4 and 5});$$

From Figure 6, $\Delta V_2 = \Delta V_3 = 820 \text{ ft/sec.}$

$$\therefore R_2 = R_3 = \exp \frac{820}{g_o (285)}$$

Applying these values to Equation (4) gives the following:

$$R_1 = \frac{K_1}{K_2 R_4 + K_3} = \frac{950}{723.49 R_4 + 79.68}$$

The same geometric construction procedure can now be applied to produce the allowable LM landing area for sortie No. 2, which is shown on Figure 4. It should be noted that the area again is elliptical in shape and that the major axis lies along a line drawn through the first and last sites visited by the LFU (sites 3 and 5). The landing area is roughly a 4 x 3.25 n mi ellipse.

TOTAL LFU MISSION ANALYSIS

Figure 5 represents the complete flying unit mission comprised of the two sorties described individually up to this point. The allowable LM landing area is shown for each of the

sorties and the overlap represents the allowable LM landing point dispersion for the total mission. If the LM lands any place within the shaded area shown, both LFU sorties can be flown as planned without either of them exceeding the 270 lb propellant limitation.

APPROXIMATE METHOD FOR DETERMINING THE ALLOWABLE LM LANDING AREA

The process described thus far is rather rigorous and time consuming, but necessary if accurate results are desired. The allowable landing footprints derived previously have been described as elliptical in shape. They are not true ellipses, however, because of the non-linear ΔV vs. range relationship (Figure 6) and due to the variations in payloads for the individual LFU hops.

If the areas are assumed to be true ellipses, location of the extreme points of each axis would be sufficient to describe the allowable landing area. An approximate method for locating these points is developed in Appendix B. There is some error in the method; however, for most practical purposes the accuracy should be sufficient. The points located by the approximate method for the two example sorties are shown on Figures 3 and 4.

CONCLUDING REMARKS

As mentioned in the earlier discussion on the limitations of the analysis procedures, an accurate LM landing will probably always be desirable for the post Apollo missions. We must remember that at least half of the mission's EVA man-hours will be spent in the LM's immediate vicinity. The main point here is to caution against interpreting the results shown in this memorandum to mean that a pinpoint LM landing will not be required.

A better interpretation would be that the mission analysis results can be used to determine the flexibility in the choice of a landing point. The results shown for the Hadley Rille mission in Appendix C illustrate this quite nicely. The allowable LM landing area determined for the proposed mission falls mostly in rough looking territory. The prospects of safe foot traverses by the astronauts would not be good in this area (see Figure C-5). Revisions to the mission's LFU sortie planning are shown to open up this landing area to include a more hospitable type terrain.

Mission planners are thus given a wider choice in the landing point selection process. Selection can be made by considering astronaut safety, scientific accomplishment, and the probability of success from the standpoint of determining the extent of the LM landing error that could be tolerated.

D. R. Valley
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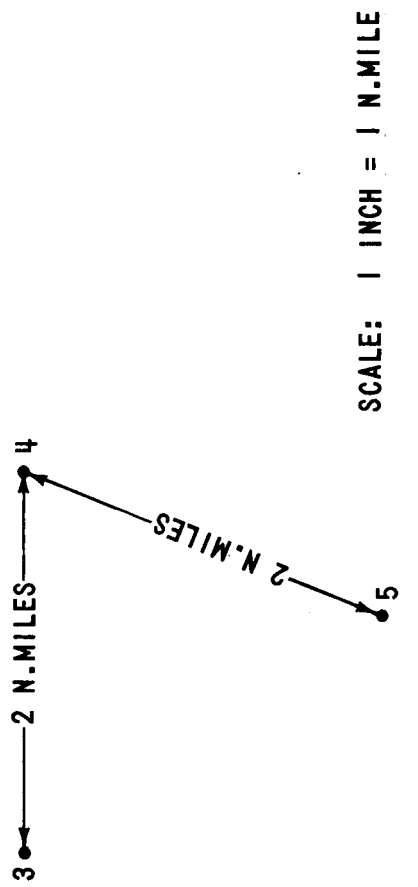
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Attachments:
References
Figures 1 to 6
Appendices A to C

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REFERENCES

1. Hadley Rille Mission presented to the Group for Lunar Exploration (GLEP) at their February 26, 1968 meeting.
2. Flight Test of a One-Man Flying Vehicle Vol. II - Final Report - Mission Applications Studies - Bell Aerosystems Report No. 2330-95002, July 1967.
3. Determination of Lunar Flying Vehicle Range and Payload Capabilities for Mission Planning Purposes, Bellcomm Memorandum For File, D. R. Valley, December 26, 1967.



PLANNED LFU SORTIES			
SORTIE NO. 1		SORTIE NO. 2	
HOP	TRAVERSE	RANGE	PLANNED PAYLOAD
1	*LM TO SITE 1	?	200 LBS.
2	SITE 1 TO SITE 2	2 N.MILES	100 LBS.
3	SITE 2 TO *LM	?	50 LBS.
1	*LM TO SITE 3	?	100 LBS.
2	SITE 3 TO SITE 4	2 N.MILES	75 LBS.
3	SITE 4 TO SITE 5	2 N.MILES	25 LBS.
4	SITE 5 TO *LM	?	25 LBS.

*LM DENOTES LM LANDING POINT

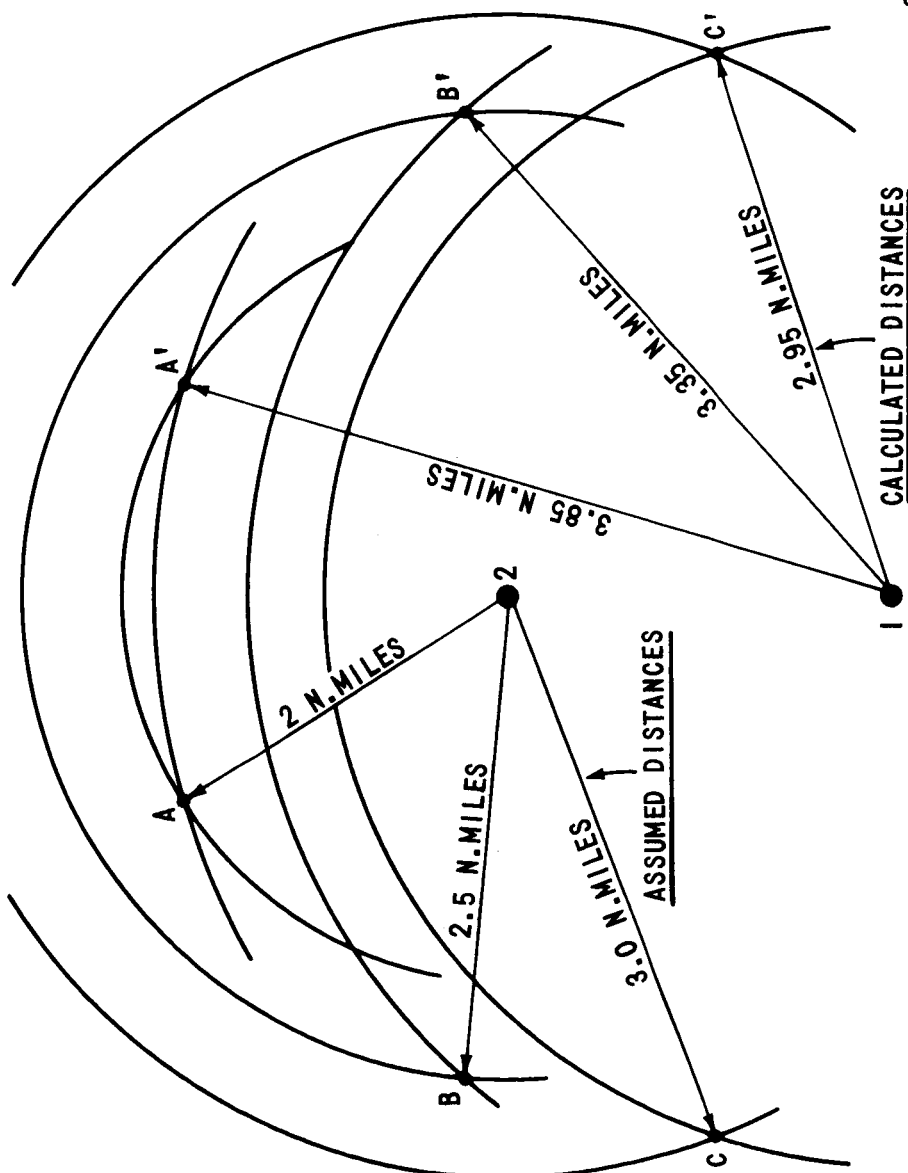
FIGURE 1 - GEOGRAPHICAL LAYOUT OF 5 SITES TO BE VISITED WITH TWO LUNAR FLYING UNIT SORTIES

STEP BY STEP PROCEDURE

1. ASSUME A DISTANCE FROM SITE 2 TO LANDING POINT.
2. CALCULATE CORRESPONDING MAX. ALLOWABLE DISTANCE FROM LANDING POINT TO SITE 1. (EQUATION 4).
3. DRAW CIRCLE CENTERED AT SITE 2 WITH RADIUS EQUAL TO DISTANCE ASSUMED IN STEP 1.
4. DRAW CIRCLE CENTERED AT SITE 1 WITH RADIUS EQUAL TO DISTANCE CALCULATED IN STEP 2.
5. THE INTERSECTIONS OF THE TWO CIRCLES OF STEPS 3&4 ARE LOCATED AT THE EXTREMITIES OF THE ALLOWABLE LM LANDING AREA.
6. REPEAT STEPS 1 THRU 5 WITH A DIFFERENT ASSUMED DISTANCE

THESE PROCEDURES SHOULD BE CONTINUED TO GENERATE THE LOCUS OF POINTS DESCRIBING THE ALLOWABLE LM LANDING AREA.

SCALE: 1 INCH = 1 N.MILE



PROCEDURES ILLUSTRATED ABOVE

<u>ASSUMED DISTANCE</u> (STEP 1)	<u>CALCULATED DISTANCE</u> (STEP 2)	<u>POINTS GENERATED TO DESCRIBE</u> <u>ALLOWABLE LM LANDING AREA</u>
2.0 N.MILES	3.85 N.MILES	A & A'
2.5 N.MILES	3.35 N.MILES	B & B'
3.0 N.MILES	2.95 N.MILES	C & C'

FIGURE 2 - GEOMETRIC CONSTRUCTION OF ALLOWABLE LM LANDING AREA FOR SORTIE NO. 1

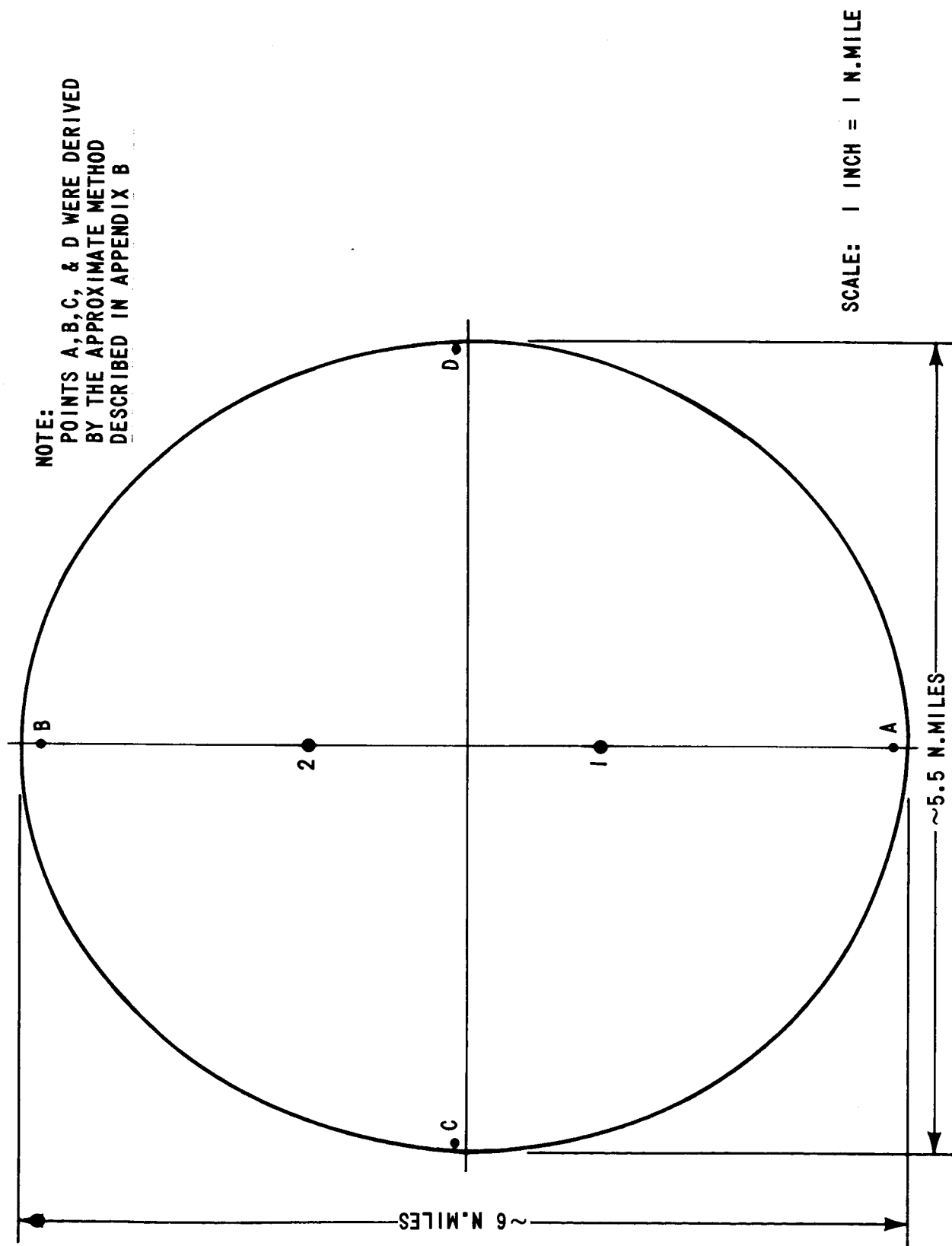
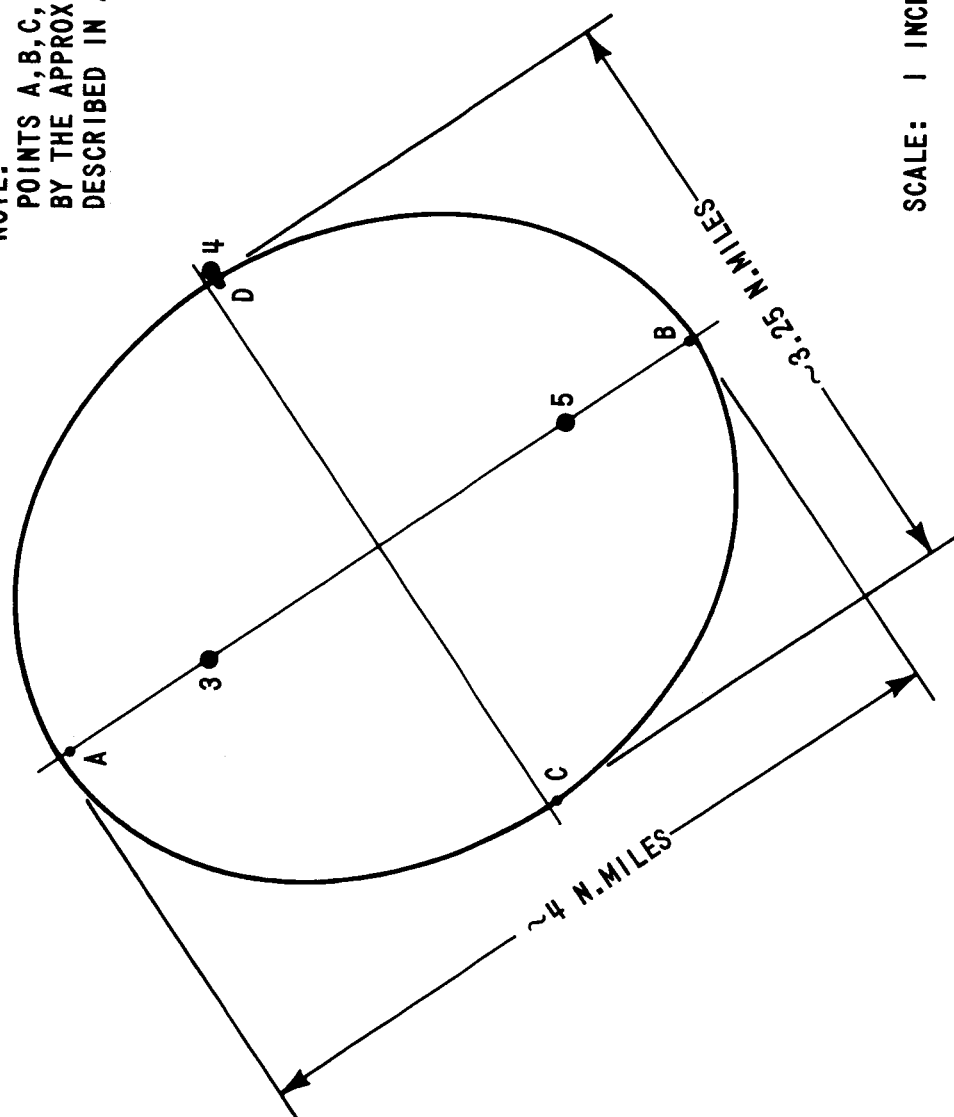


FIGURE 3 - ALLOWABLE LM LANDING AREA FOR SORTIE NO. 1

NOTE:
POINTS A, B, C, & D WERE DERIVED
BY THE APPROXIMATE METHOD
DESCRIBED IN APPENDIX B



SCALE: 1 INCH = 1 N.MILE

FIGURE 4 - ALLOWABLE LM LANDING AREA FOR SORTIE NO. 2

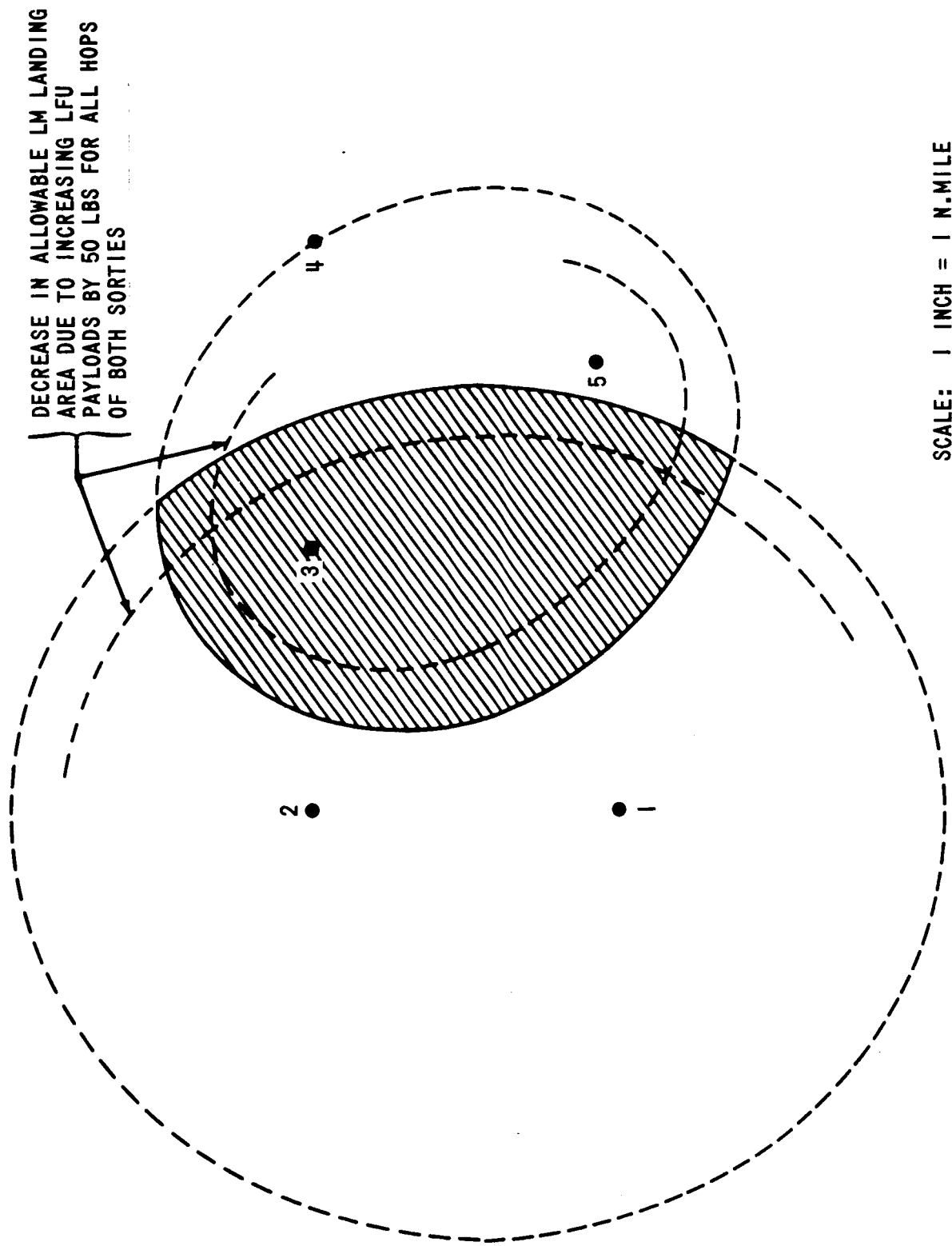


FIGURE 5 - ALLOWABLE LM LANDING AREA FOR THE TWO SORTIE MISSION

K-E 10 X 10 TO 1/2 INCH 46 1323
 7 X 10 INCHES
 MADE IN U.S.A.
 KEUFFEL & ESSER CO.

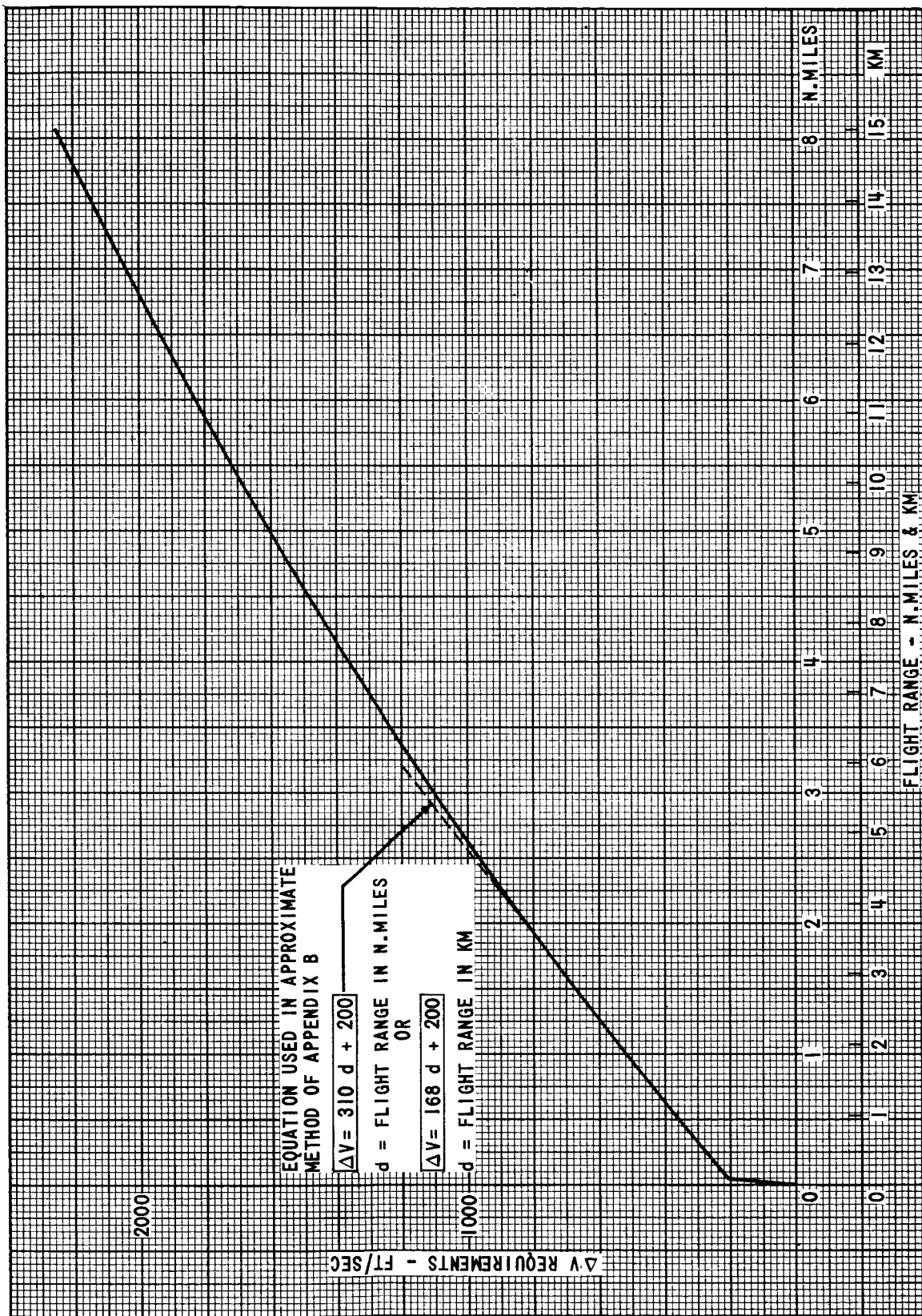


FIGURE 6 - LUNAR FLYING UNIT ΔV VS RANGE (BELL AEROSYSTEMS DATA FOR FLAT-TOP TRAJECTORY WITH A HORIZONTAL VELOCITY OF 150 FT/SEC)

APPENDIX A

DERIVATION OF AN EXPRESSION FOR THE TOTAL LFU PROPELLANT REQUIREMENTS FOR A "MULTI-HOP" SORTIE

Let:

- n = Number of hops in the sortie
- PL_1 thru PL_n = Payload weights carried on the respective hops
- wp_1 thru wp_n = LFU propellant burned on the respective hops
- R_1 thru R_n = Vehicle mass ratios associated with the ΔV requirements for the respective hops
($R = \exp \frac{\Delta V}{g_o I_{sp}}$)
- wp_t = Total LFU propellant requirements
- W = LFU burn-out weight (includes crew and propellant reserves)

First Hop

$$\text{Vehicle lift-off weight} = W + wp_t + PL_1$$

$$(1) \quad wp_1 = (W + wp_t + PL_1) \left(\frac{R_1 - 1}{R_1} \right)$$

Second Hop

$$\text{Vehicle lift-off weight} = W + wp_t - wp_1 + PL_2$$

$$(2) \quad wp_2 = (W + wp_t - wp_1 + PL_2) \left(\frac{R_2 - 1}{R_2} \right)$$

Third Hop

$$\begin{aligned} \text{Vehicle lift-off weight} &= W + wp_t - wp_1 - wp_2 + PL_3 \\ (3) \quad wp_3 &= (W + wp_t - wp_1 - wp_2 + PL_3) \left(\frac{R_3 - 1}{R_3} \right) \end{aligned}$$

nth Hop

$$(4) \quad wp_n = (W + wp_t - wp_1 - wp_2 - wp_3 - \dots - wp_{n-1} + PL_n) \left(\frac{R_n - 1}{R_n} \right)$$

The propellant requirement for each of the LFU hops must be expressed as a function of the LFU parameters (W and wp_t), the payload weights (PL) and the corresponding vehicle mass ratios (R).

$$(5) \quad wp_1 = \left(\frac{R_1 - 1}{R_1} \right) (W + wp_t + PL_1) \quad (\text{Repeat of Equation (1)})$$

$$(6) \quad wp_2 = \left[\frac{R_2 - 1}{R_2} \right] \left[\frac{W + wp_t}{R_1} - PL_1 \left(\frac{R_1 - 1}{R_1} \right) + PL_2 \right]$$

(Obtained by substituting (1) into (2) and simplifying)

$$(7) \quad wp_3 = \left[\frac{R_3 - 1}{R_3} \right] \left[\frac{W + wp_t}{R_1 R_2} - PL_1 \left(\frac{R_1 - 1}{R_1 R_2} \right) - PL_2 \left(\frac{R_2 - 1}{R_2} \right) + PL_3 \right]$$

Substituting (1) and (6) into (3) and simplifying, etc.

$$\begin{aligned} \dots \\ (8) \quad wp_n &= \left[\frac{R_n - 1}{R_n} \right] \left[\frac{W + wp_t}{R_1 R_2 R_3 \dots R_{n-1}} - PL_1 \left(\frac{R_1 - 1}{R_1 R_2 R_3 \dots R_{n-1}} \right) \right. \\ &\quad \left. - PL_2 \left(\frac{R_2 - 1}{R_2 R_3 \dots R_{n-1}} \right) - \dots - PL_{n-1} \left(\frac{R_{n-1} - 1}{R_{n-1}} \right) + PL_n \right] \end{aligned}$$

$$(9) \quad wp_t = wp_1 + wp_2 + wp_3 + \dots + wp_n$$

Adding equations 5 through 8, simplifying and solving for wp_t gives an expression for total LFU propellant requirements in terms of LFU weights, payload weights, and ΔV requirements of the individual hops.

$$(10) \quad wp_t = W \left[R_1 R_2 R_3 \dots R_n - 1 \right] + PL_1 \left[R_1 - 1 \right] + PL_2 \left[R_1 (R_2 - 1) \right] \\ + PL_3 \left[R_1 R_2 (R_3 - 1) \right] + \dots + PL_n \left[R_1 R_2 R_3 \dots R_{n-1} (R_n - 1) \right]$$

APPENDIX B

APPROXIMATE METHOD FOR DESCRIBING THE ALLOWABLE LM LANDING AREA IN CONNECTION WITH LFU SORTIES

As mentioned in the text of the memorandum, the landing footprints are elliptical in shape, but not true ellipses because of the non-linear ΔV versus range data and the vehicle weight distributions during the LFU hops. If true ellipses are assumed, however, the task of describing the area is considerably simplified over the tedious process of geometric construction used in the memorandum. The errors introduced would probably not be significant for the majority of cases.

From the symmetry apparent in the two example sorties analyzed, the major axis of the elliptical area will always lie along a line connecting the first and last sites to be visited by the LFU. Describing an elliptical landing footprint, then, simply becomes a matter of locating the points at the ends of the two axes. This appendix describes a method for locating these points.

Location of Point A (See Sketch 1 of Figure B-1)

Sketch 1 schematically illustrates the conditions required for locating one of the ellipse vertices (A). The flight ranges shown represent the distance from the landed LM to the first LFU stop (d_{1A}), the distance between the first and last site to be visited by the LFU (d_2), and the distance from the last LFU stop back to the LM (d_{3A}).

If the LM lands at point A, then:

$$d_{1A} + d_2 = d_{3A} \quad (d_2 \text{ is a known distance for a planned sortie})$$

The ΔV 's associated with the distances d_{1A} , d_2 and d_{3A} can be expressed by an equation to approximate the ΔV vs range curve of Figure 6. The equation used is shown on the figure, and its plot indicates a reasonable duplication for flight ranges in the region of interest.

Using this equation ($\Delta V = 310 d + 200$):

$$(1) \quad \Delta V_1 = 310 d_{1A} + 200 = \text{the } \Delta V \text{ associated with } d_{1A}$$

$$\Delta V_3 = 310 (d_{1A} + d_2) + 200$$

= the ΔV associated with d_{3A}

$$R_1 = \exp \frac{\Delta V_1}{u} = \exp \frac{310 d_{1A} + 200}{u} \quad (u = g_o \text{ Isp})$$

$$R_3 = \exp \frac{\Delta V_3}{u} = \exp \frac{310 d_{1A} + 310 d_2 + 200}{u}$$

$$(2) \quad R_3 = R_1 \cdot \exp \frac{310 d_2}{u}$$

Recalling equation (3) from the memorandum:

$$(3) \quad R_1 = \frac{K_1}{K_2 R_3 + K_3}$$

Substituting (2) into (3)

$$R_1 = \frac{K_1}{R_1 K_2 \cdot \exp \frac{310 d_2}{u} + K_3} \quad \text{or}$$

$$R_1^2 \left(K_2 \cdot \exp \frac{310 d_2}{u} \right) + R_1 K_3 - K_1 = 0$$

Solving the quadratic equation for R_1 (considering only positive roots since R_1 is positive by definition)

$$(4) \quad R_1 = \frac{-K_3 + \sqrt{K_3^2 + 4 K_1 K_2 \exp \frac{310 d_2}{u}}}{2 K_2 \cdot \exp \frac{310 d_2}{u}}$$

$$\Delta V_1 = g_o I_{sp} \ln R_1$$

$$d_{1A} = \frac{g_o I_{sp} \ln R_1 - 200}{310} \quad \begin{array}{l} \text{(Use of equation (1);} \\ \Delta V = 310 d_{1A} + 200) \end{array}$$

Referring to sketch 1, d_{1A} locates point A with respect to the first site.

Location of Point B (See sketch 2 of Figure B-1)

If the LM lands at point B;

$$d_{3B} = d_{1B} - d_2$$

$$\Delta V_3 = 310 (d_{1B} - d_2) + 200$$

$$\Delta V_1 = 310 d_{1B} + 200$$

$$R_3 = \exp \frac{310 (d_{1B} - d_2) + 200}{u} \quad (u = g_o I_{sp})$$

$$R_1 = \exp \frac{310 d_{1B} + 200}{u}$$

$$\frac{R_3}{R_1} = \exp \frac{-310 d_2}{u}$$

$$(5) \quad R_3 = R_1 \cdot \exp \frac{-310 d_2}{u}$$

Substituting (5) into (3)

$$R_1 = \frac{K_1}{R_1 K_2 \cdot \exp \frac{-310 d_2}{u} + K_3}$$

or

$$R_1^2 \left(K_2 \cdot \exp \frac{-310 d_2}{u} \right) + R_1 K_3 - K_1 = 0$$

Solving the quadratic equation for R_1

$$(6) \quad R_1 = \frac{-K_3 + \sqrt{K_3^2 + 4 K_1 K_2 \cdot \exp \frac{-310 d_2}{u}}}{2 K_2 \cdot \exp \frac{-310 d_2}{u}}$$

and as in the case for point A:

$$d_{1B} = \frac{g_o I_{sp} \ln R_1 - 200}{310}$$

Referring to sketch 2, the distance d_{1B} here locates point B with respect to the first site.

Location of Points C & D (See Sketch 3 of Figure B-1)

Points C & D are located such that $d_{1C,D} = d_{3C,D}$

$$(7) \quad \therefore R_1 = R_3$$

Substituting (7) into (3)

$$R_1 = \frac{K_1}{K_2 R_1 + K_3} \quad \text{or} \quad K_2 R_1^2 + K_3 R_1 - K_1 = 0$$

$$(8) \quad R_1 = \frac{-K_3 + \sqrt{K_3^2 + 4 K_1 K_2}}{2 K_2}$$

$$\therefore d_{1C,D} = d_{3C,D} = \frac{g_o I_{sp} \ln R_1 - 200}{310}$$

Points C & D will be valid points on the landing ellipse lying on the perpendicular bisector of the line connecting the first and last LFU sites to be visited during a sortie. It should be noted that these points do not necessarily lie on the minor axis, which will be midway between points A & B. The ellipse center will usually be shifted slightly toward the first or the last LFU site depending on the vehicle weight distributions during the sortie.

The amount of shift will normally be slight, and for most purposes, the distance between points C & D will be nearly equal to the minor axis of the ellipse. This distance can be determined from the right triangle shown in Sketch 4 on Figure B-1.

$$\text{Minor axis of ellipse} \approx 2\sqrt{d_1^2 - \left(\frac{d_2}{2}\right)^2}$$

where d_1 is the distance computed to locate points C & D.

To demonstrate the application of the approximate method described in this appendix, it will be applied to the two example sorties covered in the memorandum.

Approximate Method Applied to Sortie No. 1

The known parameters for this sortie can be taken from the memorandum:

$$d_2 = 2 \text{ nm}; K_1 = 1,050; K_2 = 688.93; K_3 = 154.68$$

Using these values, points A, B, C and D can be located by using equations 4, 6, and 8 from this appendix.

Point A (Sketch 1)

$$R_1 = 1.0932 \text{ from equation (4)}$$

$$\therefore \Delta V_1 = g_0 \text{Isp} \ln R_1 = 817 \text{ ft/sec}$$

$$d_{1A} = \frac{\Delta V_1 - 200}{310} = \underline{\underline{1.99 \text{ nm}}}$$

Point B (Sketch 2)

$$R_1 = 1.1625 \text{ from equation (6)}$$

$$\therefore \Delta V_1 = g_0 \text{Isp} \ln R_1 = 1381 \text{ ft/sec}$$

$$d_{1B} = \frac{\Delta V_1 - 200}{310} = \underline{\underline{3.81 \text{ nm}}}$$

Major Axis of Ellipse

$$d_{1A} + d_{1B} = \underline{\underline{5.80}} \text{ nm}$$

Points C and D (Sketch 3)

$$R_1 = 1.1274 \text{ from equation (8)}$$

$$\therefore \Delta V_1 = g_0 \text{Isp} \ln R_1 = 1,099 \text{ ft/sec}$$

$$d_{1C,D} = \frac{\Delta V_1 - 200}{310} = \underline{\underline{2.90}} \text{ nm}$$

Minor Axis of Ellipse

$$2 \sqrt{\left(d_{1C,D}\right)^2 - \left(\frac{d_2}{2}\right)^2} = \underline{\underline{5.44}} \text{ nm}$$

The size and location of this ellipse compares closely to the allowable LM landing area of Figure 3 which shows the location of points A, B, C, and D determined here.

The bulk of the error can be attributed to the above use of the equation ($\Delta V = 310 d + 200$) to convert ΔV to range. If the curve of Figure 6 is used for this conversion, most of the error is eliminated.

Approximate Method Applied to Sortie No. 2

The known parameters are:

$$d_2^* = 2.25 \text{ nm}; K_1 = 950; K_2 = 723.48; K_3 = 79.68$$

* d_2 is the measured distance between sites 3 and 5 on Figure 4.

Point A (Sketch 1)

$$R_1 = 1.0533 \text{ from equation (4)}$$

$$\therefore \Delta V_1 = g_0 \text{Isp} \ln R_1 = 476 \text{ ft/sec}$$

$$d_{1A} = \frac{\Delta V_1 - 200}{310} = \underline{\underline{0.89}} \text{ nm}$$

Point B (Sketch 2)

$$R_1 = 1.1324 \text{ from equation (6)}$$

$$\therefore \Delta V_1 = g_0 \text{Isp} \ln R_1 = 1,140 \text{ ft/sec}$$

$$d_{1B} = \frac{\Delta V_1 - 200}{310} = \underline{\underline{3.03}} \text{ nm}$$

Major Axis of Ellipse

$$d_{1A} + d_{1B} = \underline{\underline{3.92}} \text{ nm}$$

Points C and D (Sketch 3)

$$R_1 = 1.0921 \text{ from equation (8)}$$

$$\therefore \Delta V_1 = g_0 \text{Isp} \ln R_1 = 808 \text{ ft/sec}$$

$$d_{1C,D} = \frac{\Delta V_1 - 200}{310} = \underline{\underline{1.96}} \text{ nm}$$

Minor Axis of Ellipse

$$2\sqrt{\left(d_{1C,D}\right)^2 - \left(\frac{d_2}{2}\right)^2} = \underline{\underline{3.21}} \text{ nm}$$

Figure 4 of the memorandum shows the points A, B, C and D determined here.

The error introduced by the equation $\Delta V = 310 d + 200$ is not as pronounced for this sortie since the ΔV 's involved fall into a region where the equation tracks the curve of Figure 6 reasonably well.

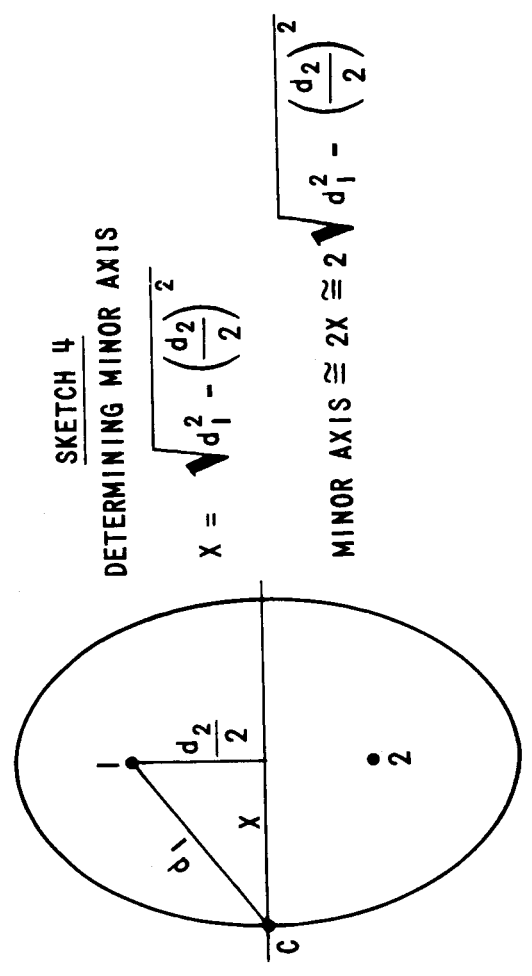
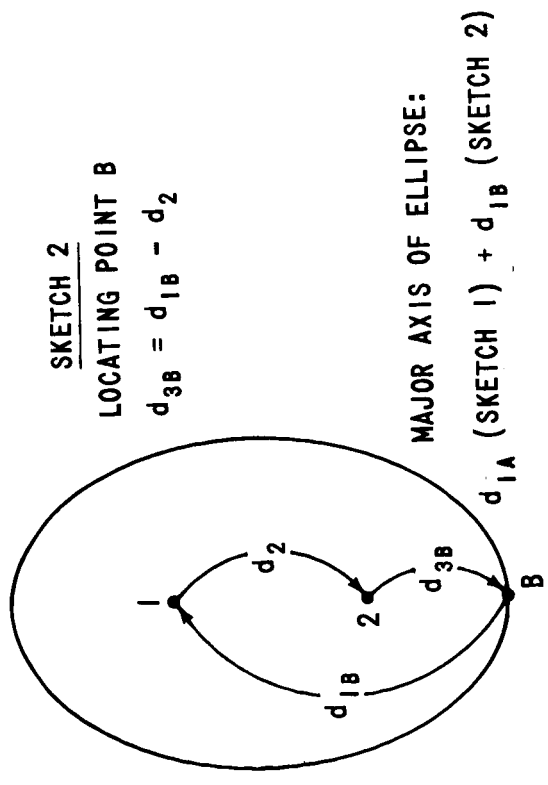
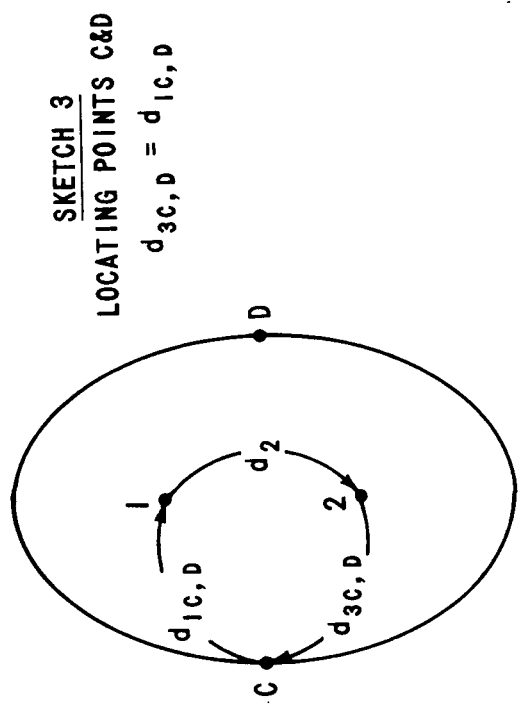
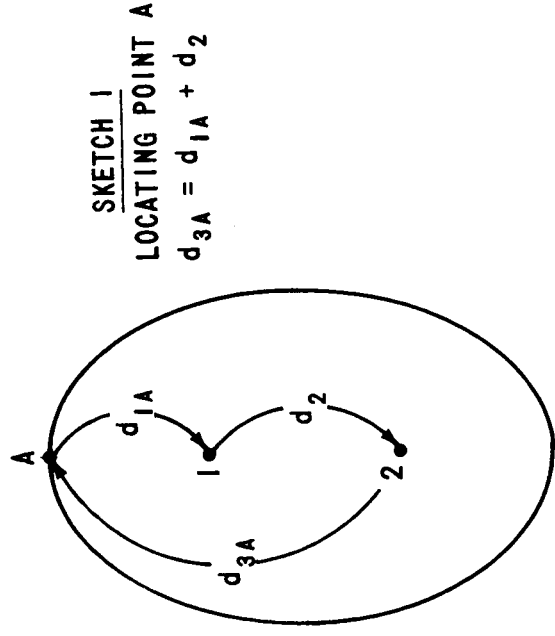


FIGURE B-1 - SCHEMATIC OF APPROXIMATE METHOD

APPENDIX C

ANALYSIS OF PROPOSED LUNAR MISSION

TO THE HADLEY RILLE REGION

The analytical methods described in the memorandum were applied to a proposed mission to the Hadley Rille region of the moon. Figure C-1 is a photograph of this region showing seven (7) selected sites to be visited via the three LFU sorties indicated; Sortie No. 1 (3-hops) to sites 1 and 2, Sortie No. 2 (3-hops) to sites 3 and 4, and Sortie No. 3 (4-hops) to sites 5, 6, and 7. The sites are to be visited in numerical sequence.

The proposed mission plan did not specify the payload weights to be carried on each LFU hop, so values were assumed for this analysis. Each sortie was presumed to start with a 200 lb. basic payload (instruments, tools, etc.) which increases by 25 lbs. for each successive hop during a sortie to allow for lunar samples picked up from each site.

The mission analysis presented in this appendix first determines the allowable LM landing point dispersion for the proposed mission using the assumed LFU payloads. Revisions to the LFU sorties are then evaluated for possible improvements to this allowable LM landing area. In all cases, the same 7 locations are visited and the same LFU payload plan is followed. The mission revisions simply alter the combinations of sites to be covered by each LFU sortie and vary the sequence in which the sites are visited during a given sortie. Figure C-2 outlines the individual sortie details for the proposed LFU mission as well as for two alternate plans (Revisions 1 and 2) that improve the allowable LM landing area. The LFU weight, performance, and flight trajectory data used are also shown on Figure C-2.

The allowable LM landing areas generated for each of the proposed LFU sorties are shown on Figure C-3. The overlap of the individual sortie footprints (shaded portion) represents the area in which the LM could land and still allow all three LFU sorties to be completed without any of them exceeding the 270 lb. propellant constraint.

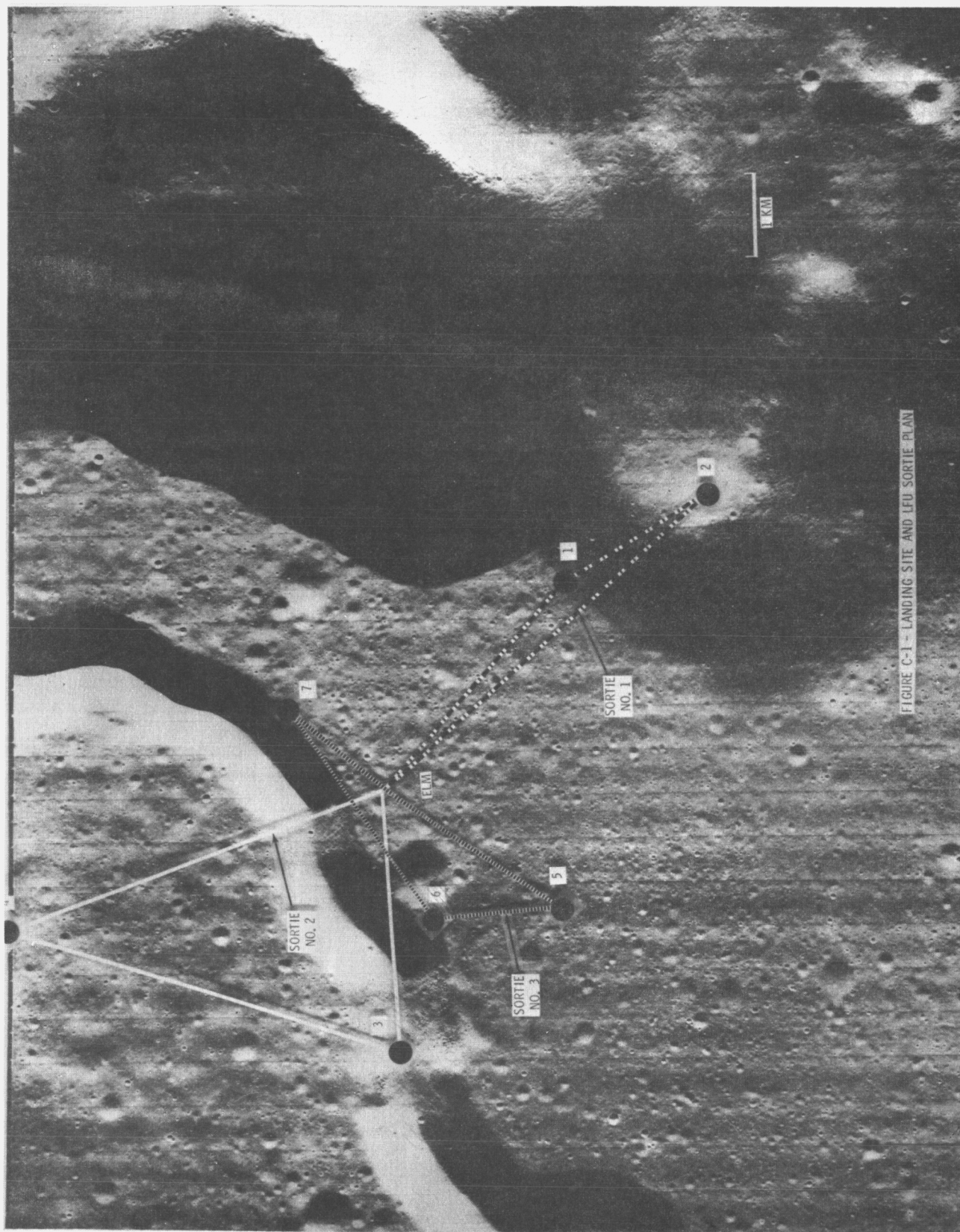


FIGURE C-1 - LANDING SITE AND LFU SORTIE PLAN

Remembering that the elliptically shaped area for each sortie will be positioned with its major axis along a line connecting the first and last site to be visited; the sites can be logically regrouped to increase the overlap area shown on Figure C-3. Revision 1 regroupes the sites so that Sortie No. 1 visits sites 1 and 2 as before, but Sortie No. 2 now goes to sites 4 and 7, and Sortie No. 3 to sites 3, 5, and 6. Revision 2 holds the same regrouping, but illustrates the effect of changing the sequence in which the sites are visited during Sortie No. 3. These revisions shift the position of the individual sortie footprints as shown on Figure C-4.

For comparison purposes, Figure C-5 shows the allowable LM landing areas for the proposed mission as well as for the revised versions superimposed on a photograph of the mission site. The relative improvements provided by the revised LFU planning are apparent from this figure. The area is not only substantially larger, but also spreads into a type terrain that appears favorable to a lunar landing.

The mission analysis shown here is intended to demonstrate the application of the techniques described in the memorandum and not as a critique of the proposed mission. The LFU payloads, for example, were selected to show a rather unfavorable landing area which includes mostly hostile terrain and, in fact, did not even contain the landing site shown in the proposed mission layout (Figure C-1). The payloads assumed, however, are not totally unrealistic and could well have been considered for this mission on the basis of planning data available at this time. The results, as presented here, point up the need for closer evaluation of the LFU capabilities for planning a mission with a high probability of success.

<u>PROPOSED MISSION</u>				<u>REVISION 1</u>				<u>REVISION 2</u>			
HOP NO.	TRAVERSE	RANGE (KM)	PAYLOAD (LBS)	HOP NO.	TRAVERSE	RANGE (KM)	PAYLOAD (LBS)	HOP NO.	TRAVERSE	RANGE (KM)	PAYLOAD (LBS)
SORTIE NO. 1	LM TO 1	-	200	1				1			
	1 TO 2	2.0	225	2				2			
	2 TO LM	-	250	3				3			
							SAME AS PROPOSED MISSION				SAME AS PROPOSED MISSION
SORTIE NO. 2	LM TO 3	-	200	1	LM TO 4	-	200	1			
	3 TO 4	4.8	225	2	4 TO 7	4.1	225	2			
	4 TO LM	-	250	3	7 TO LM	-	250	3			
											SAME AS REVISION 1
SORTIE NO. 3	LM TO 5	-	200	1	LM TO 6	-	200	1	LM TO 6	-	200
	5 TO 6	1.6	225	2	6 TO 5	1.6	225	2	6 TO 3	1.7	225
	6 TO 7	3.0	250	3	5 TO 3	2.8	250	3	3 TO 5	2.8	250
	7 TO LM	-	275	4	3 TO LM	-	275	4	5 TO LM	-	275

FLYING VEHICLE DATA

DRY WEIGHT 180 LBS.
 CREW WEIGHT 370 LBS.
 USABLE PROPELLANT 270 LBS.
 RESERVE PROPELLANT 30 LBS.
 TOTAL VEHICLE WT. (LESS PAYLOAD) 850 LBS
 SPECIFIC IMPULSE 285 SEC.
 FLIGHT TRAJECTORY - 150 FT/SEC FLAT-TOP
 (SEE FIGURE 6 OF MEMORANDUM)

FIGURE C-2 - LFU SORTIE DETAILS (FOR VISITS TO SITES 1 THRU 7)

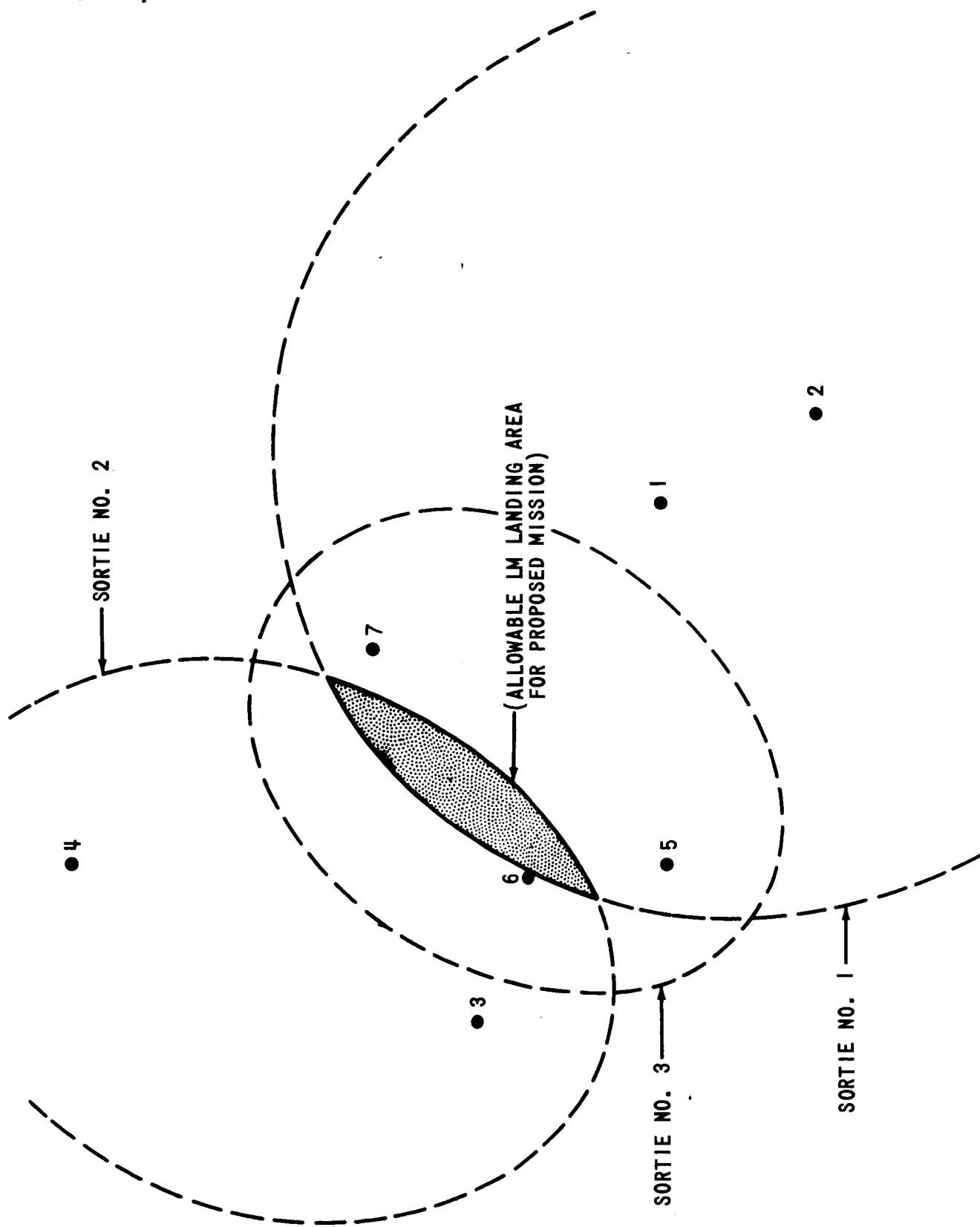


FIGURE C-3 - PROPOSED HADLEY MISSION - SORTIES 1, 2 & 3 (SCALE: 5/8 INCH = 1 KM)

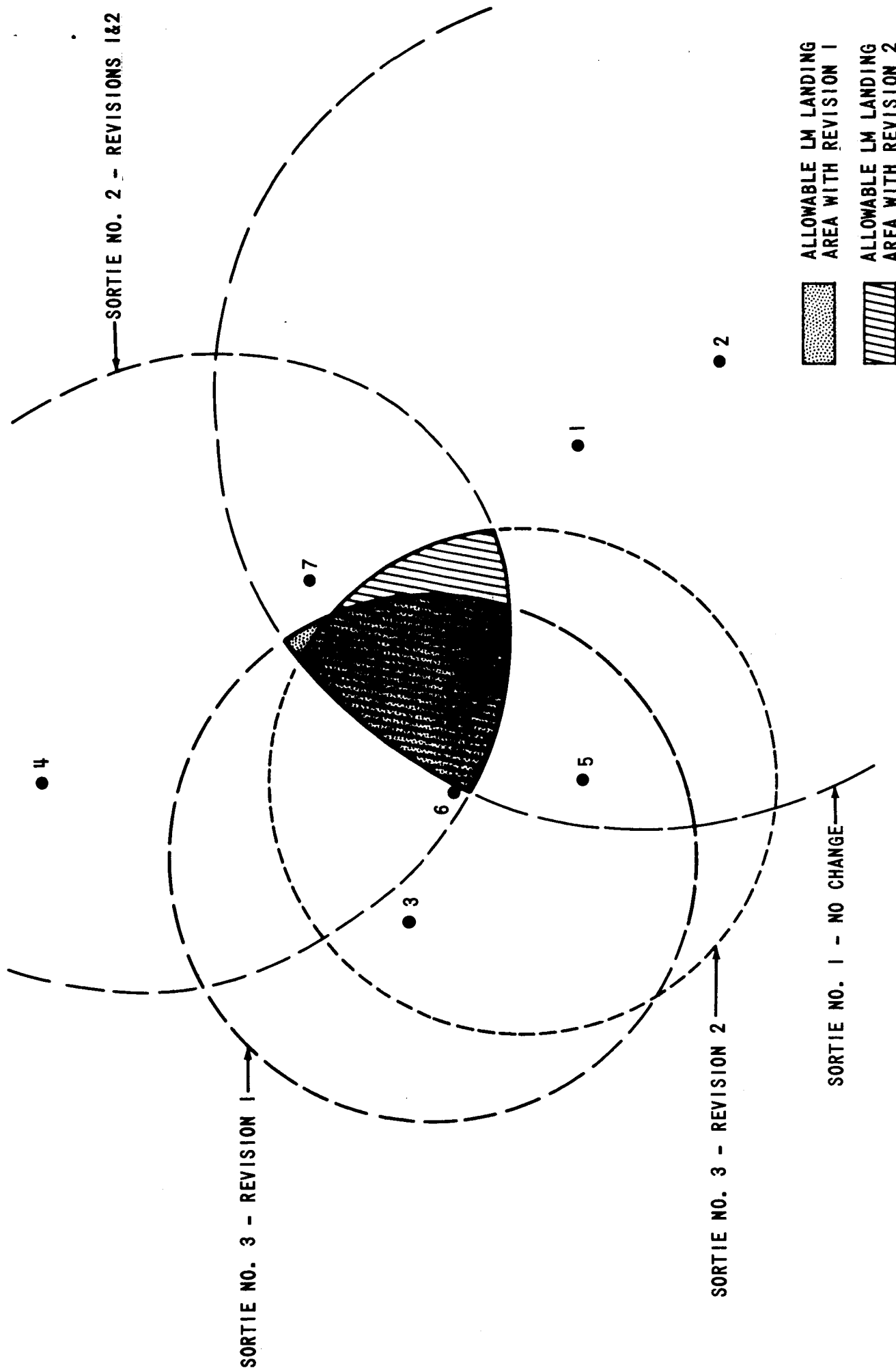


FIGURE C-4 - REVISED HADLEY MISSION (SCALE: 5/8 INCH = 1 KM)

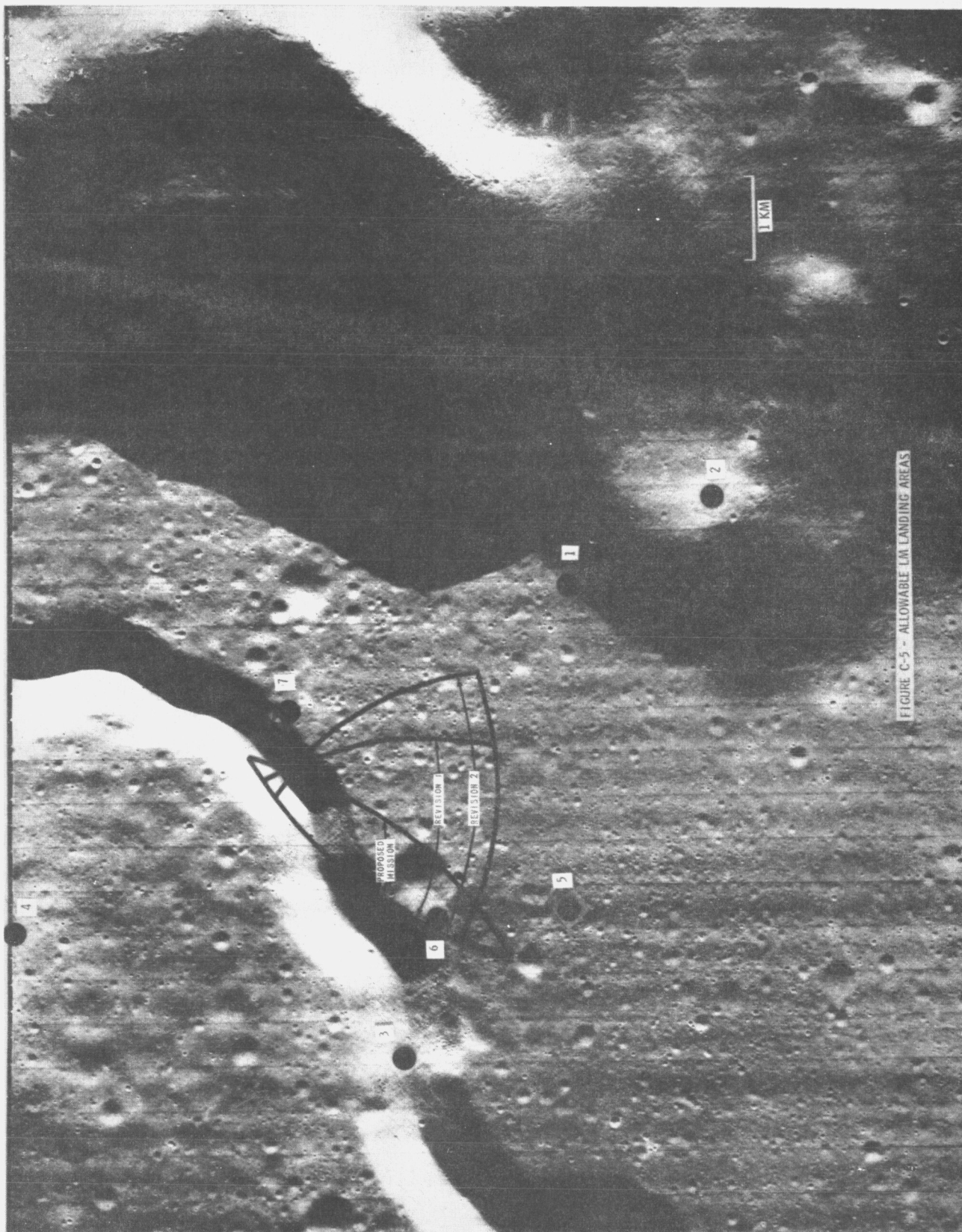


FIGURE C-5 - ALLOWABLE LM LANDING AREAS